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RISK AND UNCERTAINTY IN FOREST MANAGEMENT: CHOOSING A METHOD OF ANALYSIS

PRINICPAL INVESTIGATOR

G. C. Van Kooten

SRC Technical Report No. 219

Publication E-906-33-B-88 July, 1988

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The author wishes to acknowledge contributions made by Elaine E. Wheaton and Greg Brown although he is responsible for any errors which might remain.

SRC Publication No. E-906-33-B-88 July, 1988

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RISK AND UNCERTAINTY IN FOREST MANAGEMENT: CHOOSING A METHOD OF ANALYSIS

1. INTRODUCTION

Walters (1986) identifies "two fundamental flaws in the early development of the renewable resource sciences" (p. 2). Firstly, research and management of renewable resources, such as forest lands, have focused primarily on the biological and ecological components of the resource system, neglecting the input of social scientists, particularly economists, and ignoring the socioeconomic dynamics of such systems. Secondly, risk and uncertainty have not been adequately taken into account in decision making, nor has there been sufficient emphasis on the modeling of risk and uncertainty. The latter flaw is the focus of the current research. In particular, we seek to develop a forest management model which takes into account risk and uncertainty.

A major question that needs to be addressed is that of determining the effect of risk and uncertainty upon rotation age and annual allowable cut. In order to tackle this difficult problem, we begin in section 2 by first considering the difference between risk and uncertainty. We settle upon the following distinction: risk refers to the risk attitude of decision makers, whether they are risk neutral, risk averse or risk loving, while uncertainty refers to the stochasticity of biological, physical and economic events. Thus, models of uncertainty in renewable resource markets generally assume that the decision maker is risk neutral (i.e., that net returns are to be maximized, or costs are to be minimized), while disease, fire, prices and so on are taken to be uncertain.

Models of decision maker risk are reviewed in section 3. The literature regarding uncertainty due to supply factors--in the case of the forest, climate variability, forest fire, insect pests and disease--is also reviewed in section 3. The focus of this section will be to identify methods which can be used to model uncertainty in forest applications. In addition, we seek to determine what effect uncertainty will have on the value of the in situ resource stock and on the rate of extraction.

In section 4, we identify dynamic models of forest harvesting and methods for taking into account uncertainty in dynamic models. The discussion is quite theoretical and we examine methods for solving dynamic control problems. We focus on both deterministic and stochastic formulations of models and how these differ, and we examine methods for solving dynamic problems. We compare the Lagrangian solution (and gradient methods) with the dynamic programming approach. It turns out that, while these approaches are theoretically identical, in practice they may differ substantially.

In section 5, we examine the possibility of including the projection or Leslie matrix (Leslie 1945, 1948) in the state transformation equation. An empirical example of forest management under uncertainty is constructed in section 6, but the example is somewhat contrived as there was insufficient time to gather required data and estimate the needed relationships. Nonetheless, we compare an optimal cutting rotation in the case where there is no uncertainty with that where uncertainty is explicitly modeled. Deterministic and stochastic dynamic programming are used for the comparison. The conclusions and recommendations ensue in section 7.

2. RISK AND UNCERTAINTY

In this section, we make two separate distinctions between risk and uncertainty. The first distinction is the traditional one based on the objective versus subjective probability of the occurrence of an event. However, it is also argued that "decision makers can and should employ subjective probabilities, whether these are based on objective probabilities if they exist, or on well-considered degrees of belief" (Kennedy 1986, p.ix). Therefore, accepting this argument, we make a second distinction between risk and uncertainty for the purposes of this study. We employ the term risk to refer to the decision maker's risk attitude and the term uncertainty to refer to the occurrence of stochastic events which affect tree growth. The use of these terms is discussed in more detail below.

2.1 Objective versus Subjective Probabilities

In this subsection, we distinguish between risk and uncertainty in the following way. Risk refers to those events to which an objective probability can be assigned. This probability is based upon past observations regarding the event. Thus, it might be possible to construct objective probabilities regarding forest fire, insect pest or disease outbreak in a particular forest. Uncertainty, on the other hand, refers to those events for which it is not possible to develop objective probabilities. "Uncertainty is a state in which one alternative results in a set of possible outcomes whose probabilities are not known exactly" (Kao 1984, p.922). However, not everyone agrees with this concept of uncertainty and there is some confusion in the literature. For example, Lewis (1981) examines uncertainty in the Eastern Pacific yellowfin tuna fishery. He employs random, but

probabilistically known, transitions from one state to another (p.424).

If uncertainty is defined as a set of unknown probabilities, then an example of an uncertain event, at least from a public perception viewpoint, was the tornado which struck Edmonton in the summer of 1987, causing major damage and deaths. The probability of such an event can only be determined subjectively as there are no observations on previous tornados of this magnitude in the region. Likewise, it would be difficult to determine anything but subjective probabilities about the damage that a future tornado will cause in a particular region of the boreal forest. It may also be the case that it is not possible to construct objective probabilities regarding the possibilities of certain types of disease or insect infestation in the boreal forest.[1] Nonetheless, it is acceptable to use subjective probabilities where little or no information about past occurrences of an (uncertain) event are available. This is the Bayesian approach.

Bayesian methods are also used in adaptive dynamic programming. This technique is discussed by Walters (1986) and, at a more technical level, by Bar-Shalom (1981). Adaptive dynamic programming updates the probabilities of the occurrence of events at each stage. To our knowledge, this approach has been used in forestry only by Kao (1984). The main problem is that this technique is difficult to implement empirically. However, Kennedy has certain reservations about its use, suggesting that it may be more appropriate to simply employ stochastic dynamic programming.

> "It is likely that for the adaptive approach to be worthwhile there must be little change in the probability distribution throughout the decision periods... Parameters of an econometric forecasting model may be

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[1] For example, the boll weevil was an unanticipated event in the southern U.S. states and it destroyed many cotton plantations at the turn of the century.

updated as prices and explanatory variables are revealed, but dynamic programming is severely limited in the number of variables which can be used for revising distributions. It is also likely that for an adaptive approach to be successful the probability distribution should not have existed or remained stable for long periods before the decision periods... In both these cases, either where there is no constancy in a probability distribution, or where the constancy has been so great that there is little scope for subsequent learning, a straightforward stochastic dynamic programming approach is probably the most suitable (1986, p.86).

2.2 Uncertainty in Dynamic Optimization Models

In addition to distinguishing between risk and uncertainty, we must also consider uncertainty as a stochastic element in bioeconomic models (Clark 1976); it is necessary to distinguish between deterministic and stochastic dynamic models. Deterministic models do not include stochastic or random variables or events. For example, a deterministic model of forest management requires that forest growth be perfectly predictable in every period. A stochastic model, on the other hand, permits forest growth to vary stochastically from one period to the next. Insect pests, disease and forest fires cause timber growth from one period to the next to be random, or predictable only with some probability. The probability distribution may be determined in a classical fashion, using past occurrences of events, or the probability distribution may be determined subjectively, in a Bayesian fashion.[2]

It is uncertainty in forest growth which is the prime concern in this study. In order to study uncertainty, it is necessary to develop stochastic control models for forestry management. Continuous stochastic dynamic models

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[2]Lewis (1981) uses an a priori distribution on a random parameter in the state transformation equation. This is an example of a Bayesian approach, although he does not use adaptive DP. Rather, he employs sensitivity analysis regarding the density distribution of the parameter.

employ stochastic or Ito calculus (see Malliaris and Brock 1982; Mangel 1985) and are not used by most forest managers. However, since many foresters are already familiar with dynamic programming, a discrete form of stochastic control, namely, stochastic dynamic programming, is employed in this study. This is a reasonable approach as dynamic programming is a special way of analyzing discrete time, optimal control problems. It is also reasonable in light of the above quote by Kennedy. Since forest management decisions are made at discrete intervals, this method of formulating problems has proven useful.

3. EXPLOITATION OF A RENEWABLE RESOURCE UNDER UNCERTAINTY: A REVIEW OF THE LITERATURE

In this section, we begin by briefly reviewing methods for incorporating decision-maker risk in economic choice models. The emphasis is on methods for incorporating risk in dynamic models. This is followed by a more detailed review of stochastic optimal control models for managing renewable resources such as forests.

3.1 Review of Decision Maker Risk in Economic Models

When risk is present, models which have as an objective the maximization of expected profit or expected return have not performed very well in predicting the behaviour of decision makers. Therefore, economists have turned to alternative specifications of the objective function. In particular, maximization of expected utility has enjoyed a fair amount of success (Anderson, Dillon and Hardaker 1977; Lewis 1981; Barry 1984), although there is evidence that this approach is not a panacea for decision making (Binswanger 1980; Schoemaker 1982). The main problem has been that of identifying the appropriate shape of the utility function and problems of replication have been encountered. Conclusions reached in one study are not the same as those of another, similar study, even where the same sample is employed.

Others have favored the use of safety-first models as these appear to have greater appeal, at least in agriculture, for extension work (Roy 1952; Telser 1955-56; Katoaka 1963; Pyle and Turnovsky 1970; Young 1979; Musser, Ohannesian and Benson 1981). Yet others have evaluated alternative decisions on the basis of the second- and higher-order moments of the

outcomes. That is, rather than simply examining the expectation of net returns, the variance of net returns is also included in the decision regarding which alternative is preferred. The approach is to simulate a variety of management strategies, determining the expected value (E) and variation (V) of each strategy, and, then, constructing an E-V production frontier. Using risk elicitation procedures, a utility function for the decision maker can be also be constructed in E-V space (Anderson, Dillon and Hardaker 1977). The optimal management strategy is then the one which enables the decision maker to attain the highest possible level of utility. This method and sophisticated modifications of it (e.g., stochastic dominance) are frequently used to model agricultural decisions in a risk environment (Anderson, Dillon and Hardaker 1977; Brown 1987).

There have been few attempts to incorporate risk aversion on the part of decision makers in dynamic decision models. Including a decision maker's risk attitude in dynamic optimization models is a difficult task, and there has been only cursory work in this direction. One reason for this is simply that it is difficult to adequately model risk attitudes in static models let alone dynamic ones. The problem is that of determining the shape of the utility function for the decision maker. A literature search indicated that there have been limited attempts to model risk attitudes in a dynamic forest model (see Johansson and Lofgren 1985, Chapter 5 and section 3.2 below). To our knowledge, work on modeling of risk attitudes in dynamic resource models has taken place primarily in agriculture.

One method for incorporating risk attitudes in dynamic models is similar to that suggested above, namely, to determine the expected value and variance of a number of alternative strategies over time. Once the E-V frontier for

all possible strategies has been determined it is necessary to find the utility function for the decision maker. Again the chosen strategy is the one where the highest indifference curve is tangent to the E-V frontier. Burt and Johnson (1967) use this criterion to identify a risk-efficient combination of a given collection of dynamic programming strategies. The result is a risk-efficient or diversified set of strategies. However, this method does not incorporate risk preferences directly in the optimizing framework and, therefore, is difficult to use in applied work. Once again, the major obstacle is that of identifying the decision maker's utility function.

Karp and Pope (1984), on the other hand, include risk preference directly in a dynamic optimization model for range management. Although they consider both risk-neutral and risk-averse decision makers, they do not attempt to illicit this type of information from the decision maker. Indeed, Karp and Pope do not attempt to provide anything but a numerical illustration of their model. Their method of identifying the single, risk-sensitive strategy again requires knowledge of the decision maker's utility function. As noted above, in practical settings, it may be difficult and costly to determine the (risk) utility functions of individuals. Further, once these have been found, replication appears to be a problem as noted above.

However, there remains a question regarding the validity of the procedure employed by Karp and Pope. Suppose we are interested in maximizing consumption over two periods, given that income in period 1 is known but period 2 income is uncertain. It will make a difference to the optimal consumption in period 1 (and, hence, period 2) whether the outcome of the lottery is known now or at the end of the first period. The directional

change on period 1 consumption will depend upon the shape of the inter-temporal utility function. That is, the independence condition of the expected utility model is violated even in a simple two-period (expected) utility maximization model as it depends upon the time at which uncertainty is revealed (Mossin 1969). Indeed, the economics literature indicates that temporal resolution of uncertainty in the context of risk averse behaviour is not straightforward (Kreps and Porteus 1978; Spense and Zeckhauser 1972). It appears that either the inter-temporal utility function does not exist or, if it does exist, it is impossible to determine its shape. It is not clear, therefore, that expected utility maximization is an appropriate and correct procedure in dynamic models.[3]

Van Kooten and Young (1988) incorporate actual decision maker risk into a dynamic programming model for crop rotations.[4] Their method relies on a safety-first criterion suggested by Katoaka (1963), which has been shown to be equivalent to expected utility maximization under certain restrictive assumptions (Pyle and Turnovsky 1970). At this stage, however, the empirical application of the model is quite simple in terms of the number of control variables, although research is currently underway to increase the size of the problem. One drawback of the Van Kooten and Young model is that it is for an annual decision process, while the Karp and Pope model is for a renewable resource in the long run. Although the rangeland example is closer

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[3]Karp and Pope, for example, are unclear about how they specify their cardinal utility function.

[4]Van Kooten and Young found that agricultural producers in the dryland cropping district of eastern Washington, known as the Palouse region, tended to increase their summerfallow as they became more risk averse. Increased summerfallow results in much greater soil erosion as the slopes in this region are very steep.

to that of the forest, in practice, the procedure employed by Karp and Pope also relies upon an annual decision process. However, the authors do not feel that their approach to decisions under risk in a dynamic framework suffers from the criticisms leveled at expected utility maximization.

3.2 Uncertainty in Dynamic Models: Studies Using Stochastic Control

Stochastic optimal control theory is the appropriate tool for analyzing problems of uncertainty in forest management. Stochastic control theory is used to examine problems in economics and finance (Malliaris and Brock 1982), to analyze uncertainty in natural resource systems (Pindyck 1980, 1984), and to determine optimal rotation ages in the presence of catastrophic fires (Reed 1984; Reed and Clark 1987). However, not only are solutions to stochastic control models difficult to derive analytically, but numerical solutions are also difficult to obtain. In applied settings, there have been three approaches. (1) Stochastic models have been ignored in favour of deterministic ones, with sensitivity analysis used to investigate alternative scenarios. (2) The solution to the deterministic problem is used in a feedback fashion in a stochastic environment (Chow 1975). (3) Stochastic dynamic programming is employed (Lewis 1981). While the first approach is generally regarded as inferior, it is not clear whether (2) or (3) is more appropriate. However, this issue is discussed in a later section.

In the opinion of the current author, more has been written in the economics literature about uncertainty pertaining to exhaustible resources than to renewable resources, particularly if the fisheries literature is ignored. The reason for this fascination with exhaustible resources is the uncertainty about oil (energy) prices due to the oil shocks of 1973 and 1979,

and the subsequent fall in prices due to the partial collapse of OPEC resulting from the general world-wide recession. Thus, demand uncertainty is one area that resource economists have examined in moving from deterministic to stochastic dynamic optimizing models. In addition, discoveries of new reserves in the North Sea, Arctic and off-shore Newfoundland have focused attention on uncertainty regarding reserves. Although much has been written, we cite papers by Pindyck (1980) and by Arrow and Chang (1982). Arrow and Chang examine a model in which discrete increments of reserve discoveries occur stochastically as a function of exploration level. The stochastic process follows a Poisson process. Pindyck considers a model with both demand (price) uncertainty and uncertainty with regards to the level of reserves. The stochastic processes are Brownian so that Ito calculus is required and the control variables are consumption and exploration activity. Both papers examine the impact of uncertainty on prices and exploration, with exploitation considered to be the result of these.

Routledge (1980) considers the case of catastrophic mortality (primarily due to forest fire) on the optimal forest rotation policy. The Faustmann approach indicates that the optimal policy is to let the forest grow as long as

$$(R_{t+1} - R_t)/R_t > i\ (1 + L/R_t),$$

where i is the discount (interest) rate, R_t is the value of the forest t years after it was last cut and L is the soil expectation or the value of the land once it is clearcut--the optimal discounted value of all future returns. The optimal rotation age T occurs at the time when the inequality disappears.

Uncertainty in forest growth implies that "a stand should be harvested as soon as the expected growth in value of the standing trees no longer

exceeds the expected loss in interest incurred by postponing this and all subsequent harvests by 1 year" (1980, p.391). Catastrophic mortality affects the Faustmann formula, and the new rule is to cut the forest as soon as

$$R_{t+1} - R_t \leq i\,(R_t + L_0) + p(t)\,(1 - k_{t+1})\,R_{t+1},$$

where $L_0 < L$ is the mean value of clearcut land that is adversely affected by catastrophe, $p(t)$ is the probability that a catastrophe affects a stand aged t in the next cutting cycle, and k_{t+1} is the expected proportion of the stand that can be salvaged if catastrophe struck a stand of age t+1.

The effect of $p(t)\,(1 - k_{t+1})\,R_{t+1}$ is to reduce the optimal rotation age, while the effect of L_0, since it is less than L, is to increase the optimal rotation age. Routledge uses a numerical example to indicate that the former effect tends to dominate, indicating that the net effect of uncertainty due to catastrophe is to reduce the optimal rotation age. That is, the forest should be cut earlier under uncertainty compared to the time suggested by deterministic models.

The identical result was proven mathematically by Pindyck (1984) using Ito's Lemma for the more general case of renewable resource markets. Pindyck was interested not only in the value of the stock, as was Routledge, but he was also interested in harvesting or extraction costs. His main conclusions are as follows.

> "Stochastic fluctuations will thus affect the rate of extraction in three different ways: (i) Because fluctuations reduce the value of the stock, and because their variance is an increasing function of the stock level, there is an incentive to reduce the stock level by harvesting faster. (ii) Fluctuations increase expected extraction costs over time, and this also creates an incentive to extract at a faster rate. (iii) Given a fixed extraction rate, at any stock level x fluctuations reduce the expected growth rate of the stock, and this in turn reduces the extraction rate. Given a particular current stock level x, the net effect of uncertainty on the current rate of extraction is therefore indeterminate" (1984, p.294).

Reed (1984) and Reed and Errico (1985) also consider the case of forest fires. They assume that the probability of fire is independent of the age of the trees (which clearly contradicts findings by Clark, Jones and Holling (1977), Ludwig, Jones and Holling (1978), and Holling (1986)). Reed finds that, "as far as the optimal cutting age is concerned, the only consequence of a constant risk of fire is to add a premium to the discount rate" (Reed and Errico 1985, p.685). Reed and Errico (1985) are only interested in graphically demonstrating that the optimal rotation age to achieve maximum long-run average yield is reduced as a result of forest fire risk. They do indicate that, for forests in the Fort Nelson, B.C. area, the optimal rotation age is reduced from 105 years to 93 years as a result of a fire rate of 0.01.

Reed and Errico (1986) use linear programming to solve a stochastic optimal control model of fire risk. They replace the random matrices in the transformation equation with their expected values, thereby giving a deterministic model. Under certain conditions, the optimal solution to the deterministic model is an approximately optimal solution to the stochastic control problem. Assuming that the terms in the variance-covariance matrix are small, the solution to the deterministic problem can then be used in a feedback fashion within the stochastic environment. This provides the solution to the stochastic control problem.

Lewis (1981) employs a Markov decision process model (stochastic dynamic programming) to describe optimal management strategies for an ocean fishery under uncertainty. Uncertainty is generated by constructing a uniform density distribution on the catch parameter and making price a random

variable.[5] Lewis finds that the decisions obtained from deterministic models of resource management can be significantly different from those reached using a stochastic model. For example, he finds that the difference between net discounted returns for the stochastic and the deterministic problems can increase quite dramatically as the variation in the random catch parameter increases. It is clear that under uncertainty the optimal decisions are quite different from those in the deterministic case. Lewis also shows how a risk averse policy differs from a risk neutral policy, where risk is determined by the curvature of the decision maker's utility function. In particular, the risk neutral decision maker will employ greater effort at higher population levels, but less effort at lower levels of the population.

Lembersky and Johnson (1975) employ an infinite-horizon Markov decision process to investigate optimal policies for managed forests stands. In the application of their stochastic control model, they employ 240 state variables and 15 decision alternatives. However, the authors do not attempt to compare their results with those which would hold under deterministic conditions.

Johansson and Lofgren (1985) consider the case of uncertainty and its impact on the optimal forest rotation. They prove that, under certain restrictive conditions, "the stochastic case does not provide any information about the expected optimal rotation period compared with the optimal rotation in the deterministic case" (p.264).[6] However, upon relaxing the restrictive assumptions, they come to similar conclusions as Routledge

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[5]It is not clear what probability density function is employed for price, although it would appear that it is based on historic data.

[6]They argue that this result is also contained in Brock, Rothschild and Stiglitz (1979), although this paper is generally unavailable.

(1980) and Pindyck (1984).

Johansson and Lofgren (1985) also discuss a model for including risk in the harvest decision. As with Lewis (1981), this model is based on an expected utility function for woodlot owners. (The authors deal with forest management in Sweden.) Their results can be summarized as follows (pp.268-275). (1) In a certain environment, an increase in today's timber price will increase cutting today, but in an uncertain environment cutting levels are affected in an ambiguous way. (2) An increase in tomorrow's timber price relative to today's will decrease today's cutting (increase the rotation length) both in the deterministic and stochastic models. (3) A mean preserving increase in the variability of future net timber prices will increase today's cutting. (4) A lump-sum tax will increase today's cutting in an uncertain environment, but will not affect cutting in a deterministic environment. (5) Likewise, an increase in the proportional tax on profit will increase today's cutting if future prices are uncertain, but will leave things unchanged in a certain world.

4. DYNAMIC FOREST MODELS FOR ANALYZING UNCERTAINTY

In this section, we develop a theoretical management model for the boreal forest. A deterministic version of the model is first developed. It is then extended to investigate the occurrence of uncertain events (e.g., insect pests and forest fires). A discussion of other dynamic models employed in forest management can be found in Bowes and Krutilla (1985) and Reed (1986).

We begin in section 4.1 by developing a deterministic model which has some similarity to the one developed by Lyon and Sedjo (1983). The deterministic model is solved analytically to illustrate some important economic and physical requirements for an optimum, and to provide a basis for solution algorithms which is the topic of section 4.2. In section 4.2, we examine the gradient method, dynamic programming and the approximately optimal decision rule as approaches for solving the dynamic model. We also indicate that the Lagrangian method used in section 4.1 is identical to the solution derived using analytical dynamic programming (section 4.2.2).

Finally, in section 4.3, we discuss stochastic optimal control methods and consider discrete-time, stochastic dynamic programming as a method for analyzing uncertainty in forest management. An empirical application of the deterministic and stochastic DP models is provided in section 5, where the benefit function is also discussed in more detail.

4.1 A Deterministic Forest Management Model

The number of acres of timber of age j+1 at time t+1, $x_{j+1,t+1}$, will depend upon the number of years since regeneration (since last harvest), the number of acres of timber of age j in the previous year (x_{jt}), and the number

of acres of timber of age j harvested.

(1) $x_{j+1,t+1} = x_{jt} - u_{jt} x_{jt} = (1 - u_{jt}) x_{jt}$, (j=0, ..., A-1; t=0, ..., T-1)

where u_{jt} is the proportion of acres of age j harvested in year t. However, the number of acres of timber of age 1 year is equal to the number of acres harvested in the previous year; that is,

$$(2) \qquad x_{1,t+1} = \sum_{j=1}^{A} u_{jt} x_{jt}, \quad t = 0, \ldots, T-1,$$

Further, the number of acres of trees of age A (the maximum age trees are assumed to attain) at time t+1 is equal to the number of acres of trees of age A at time t plus the number of acres of age A-1 at time t, minus the number of acres of ages A and A-1 harvested at time t.[7] Thus, it is simply assumed that trees of age A which are left standing remain at age A. Hence, any characteristics which describe trees of age A also describe these trees the following year.

Let m_{jt} be the management decision regarding trees of age j at time t. At time t, a management decision is made regarding the trees harvested in that period; we denote this decision by m_{0t}. This decision may be to plant 2-year old seedlings, seed the area or do nothing. Likewise, management decisions are also made about all trees of "age" j at time t. These decisions may include chemical spraying for insect pests or expenditures on forest fire prevention. In the deterministic model, management decisions are assumed to increase timber growth,with the decision represented by the level of expenditure.

The volume of commercial timber of "age" j available at time t+1,

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[7]It is assumed that trees cannot be older than A years of age. While this is obviously not true, it is likely that trees beyond a certain age simply do not grow bigger.

$v_{j+1,t+1}$, measured on a per unit area basis (e.g., per acre) is a function of the volume of timber growing on that site the previous year (v_{jt}) and the management characteristics employed on those trees in the previous time period (m_{jt}); that is,

$$(3) \qquad v_{j+1,t+1} = f(v_{jt}, m_{jt}),$$

where $f(\ )$ can be considered a production function for timber. It is likely that timber growth increases at an increasing rate over some range of ages, and then increases at a decreasing rate. It is also assumed that growth increases with management, but at a decreasing rate: $\delta f/\delta m_{jt} > 0$ and $\delta^2 f/\delta m_{jt}^2 < 0$.

To find the volume of timber from a particular age category, it is necessary to multiply the production function by the number of acres of trees of this age. Then total volume is obtained by summing over all age categories. The total volume of commercial timber that is harvested in a given year is denoted by H_t and is equal to the number of acres of each "age" category harvested multiplied by the corresponding production function; that is,

$$(4) \qquad H_t = \sum_{j=0}^{A} u_{jt}\, x_{jt}\, f(v_{jt}, m_{jt}), \quad t = 0, \ldots, T-1.$$

The objective function is the present value of the infinite stream of net returns $R_t(H_t, u_{jt}, m_{jt})$ from timber harvesting over the planning horizon ($t = 0, \ldots, T$):

$$(5) \qquad \max_{u_{jt}, m_{jt}} \sum_{t=0}^{T-1} [R_t(H_t, u_{jt}, m_{jt})]\, \beta^t + \Omega(x_{jT})\, \beta^T,$$

where $\Omega(\ )$ is the (salvage) value of the forest at the end of the planning horizon and β is the discount rate. Problem (5) is maximized subject to

Equations (1), (2), (3) and (4). In addition, there is a set of initial conditions for the number of acres in each "age" group and there are constraints on the values which the variables can take. These constraints are written, respectively, as:

(6) $x_{j0} = a_{j0}$ for all $j = 0, \ldots, A$; and

(7) $0 \leq u_{jt} \leq 1$, $x_{jt} \geq 0$ and $u_{0t}=0$.

It is assumed that no timber of "age" 0 is harvested.

The Lagrangian for the constrained maximization problem is:

$$(8) \quad L = \sum_{t=0}^{T-1} [R_t(H_t, u_{jt}, m_{jt})] \beta^t + \Omega (x_{jT}) \beta^T,$$

$$- \lambda_{1,t+1} \beta^{t+1} [x_{1,t+1} - \sum_{j=1}^{A} u_{jt} x_{jt}]$$

$$- \lambda_{j+1,t+1} \beta^{t+1} [x_{j+1,t+1} - (1 - u_{jt}) x_{jt}]$$

$$- \theta_{j+1,t+1} \beta^{t+1} [v_{j+1,t+1} - f(v_{jt}, m_{jt})]$$

$$- \gamma_t \beta^t [H_t - \sum_{j=0}^{A} u_{jt} x_{jt} f(v_{jt}, m_{jt})]$$

$$- \phi_j (x_{j0} - a_{j0}).$$

The Lagrange multipliers are λ_{rs}, $\theta_{r,s-1}$, γ_s and ϕ_r, where $0 \leq r \leq A$ and $0 \leq s \leq T$. Values of the Lagrangian multipliers outside this range are zero.

The first-order conditions for a maximum of the Lagrangian are as follows:

(9) $\delta L/\delta H_t = \delta R_t/\delta H_t \beta^t - \gamma_t \beta^t = 0, \quad t = 0, \ldots, T-1;$

(10) $\delta L/\delta x_{jt} = - \lambda_{jt} \beta^t + \lambda_{1,t+1} \beta^{t+1} u_{jt} + \lambda_{j+1,t+1} \beta^{t+1} (1 - u_{jt})$
$+ \gamma_t \beta^t u_{jt} f(v_{jt}, m_{jt}) = 0, \quad t=0, \ldots, T-1;$

(11) $\delta L/\delta x_{j0} = - \lambda_{j0} + \lambda_{j+1,1} \beta (1 - u_{j0}) + \gamma_0 u_{j0} f(v_{j0}, m_{j0}) - \phi_j = 0;$

(12) $\delta L/\delta x_{jT} = (\delta \Omega_T/\delta x_{jT}) \beta^T - \lambda_{jT} \beta^T = 0;$

$$(13)\quad \delta L/\delta v_{jt} = -\theta_{jt}\,\beta^{t} + \theta_{j+1,t+1}\,\beta^{t+1}\,\delta f/\delta v_{jt} + \gamma_t\,\beta^{t}\,u_{jt}\,x_{jt}\,\delta f/\delta v_{jt} = 0,$$
$$t=0,\ \ldots,\ T-1;$$

$$(14)\quad \delta L/\delta u_{jt} = \lambda_{1,t+1}\,\beta^{t+1}\,x_{jt} - \lambda_{j+1,t+1}\,\beta^{t+1}\,x_{jt}$$
$$+ \gamma_t\,\beta^{t}\,x_{jt}\,f(v_{jt},\ m_{jt}) = 0,\quad t=0,\ \ldots,\ T-1;$$

$$(15)\quad \delta L/\delta m_{jt} = (\delta R_t/\delta m_{jt})\,\beta^{t} + \theta_{j+1,t+1}\,\beta^{t+1}\,\delta f/\delta m_{jt}$$
$$+ \gamma_t\,\beta^{t}\,u_{jt}\,x_{jt}\,\delta f/\delta m_{jt} = 0,\quad t=0,\ \ldots,\ T-1.$$

Notice that, in the foregoing optimization problem, there are two counters--one for time and another for "age" of trees. The problem can, however, be reduced to a single counter by using matrices. Define the following vectors and matrices:

$$X_t = \begin{bmatrix} x_{0t} \\ \cdot \\ \cdot \\ \cdot \\ x_{At} \end{bmatrix}_{(A+1)\ X\ 1} \qquad U_t = \begin{bmatrix} u_{0t} & \cdots & 0 \\ \cdot & & \cdot \\ \cdots & & \cdots \\ \cdot & & \cdot \\ 0 & \cdots & u_{At} \end{bmatrix}_{(A+1)\ X\ (A+1)}$$

$$K = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & .. & 1\ \ 0\ \ 0 \\ 0 & 0 & .. & 0\ \ 1\ \ 1 \end{bmatrix}_{(A+1)\ X\ (A+1)} \qquad B = \begin{bmatrix} 0 & -1 & -1 & \cdots & -1 \\ 0 & 1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{(A+1)\ X\ (A+1)}$$

$$F_t = \begin{bmatrix} f_{00}^{t} & \cdots & f_{0A}^{t} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ f_{A0}^{t} & \cdots & f_{AA}^{t} \end{bmatrix}_{(A+1)\ X\ (A+1)} \qquad V_t = \begin{bmatrix} v_{0t} \\ \cdot \\ \cdot \\ v_{At} \end{bmatrix}_{(A+1)\ X\ 1}$$

$$J = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(A+1)\ X\ (A+1)} \qquad M_t = \begin{bmatrix} m_{0t} \\ m_{1t} \\ \vdots \\ m_{At} \end{bmatrix}_{(A+1)\ X\ 1}$$

$X_0 = [a_0\ a_1\ \cdots\ a_A]_{1X(A+1)}$

where F_t represents the production function and X_0 is a vector of the initial distribution of acres in the forest according to "age" (i.e., number of years since last regeneration).[8]

Using matrix notation, Equations (1) and (2) can be represented by a single equation of motion, namely,

(16) $\quad X_{t+1} = K\ X_t - B\ U_t\ X_t.$

Similarly, in matrix notation, the equation of motion for growth in commercial timber volume (3) becomes

(17) $\quad V_{t+1} = F_t(V_t, M_t),$

while the initial values are:

(18) $\quad X_0 = [a_i],\ i = 0, \ldots, A.$

In matrix notation, the harvest function becomes:

(19) $\quad H_t = U_t\ F_t\ X_t\ e.$

Finally, the requirement that $u_{0t} = 0$ is included directly in the model as the values of the first row and first column in both the B and U_t matrices are set equal to zero.

The objective function now becomes $R_t(H_t, U_t, M_t)$. Hence, the problem (5) is reduced to the following:

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[8]Last regeneration could have occurred as a result of forest fire.

$$(20) \qquad \underset{U_t, M_t}{\text{Max}} \sum_{t=0}^{T-1} R_t(H_t, U_t, M_t)\, \beta^t + \Omega(X_T)\, \beta^T$$

$$\text{s.t. } X_{t+1} = K X_t - B U_t X_t,$$

$$V_{t+1} = F_t(V_t, M_t),$$

$$H_t = U_t F_t X_t, \text{ and}$$

$$X_0 = [a_i].$$

The state variables are X and V, while the control variables are U and M.

The Lagrangian associated with this problem is similar to (8):

$$(21) \qquad L = \sum_{t=0}^{T-1} R_t(H_t, U_t, M_t)\, \beta^t + \Omega(X_T)\, \beta^T$$

$$- \lambda_{t+1}\, \beta^{t+1} [X_{t+1} - (K X_t - B U_t X_t)]$$

$$- \theta_{t+1}\, \beta^{t+1} [V_{t+1} - F_t(V_t, M_t)]$$

$$- \gamma_t\, \beta^t (H_t - U_t F_t X_t).$$

The first-order conditions for a maximum are:

$$(22) \quad (\delta R_t/\delta H_t)\, \beta^t - \gamma_t\, \beta^t;$$

$$(23) \quad (\delta R_t/\delta U_t)\, \beta^t - \lambda_{t+1}\, \beta^{t+1} B X_t + \gamma_t\, \beta^t F_t X_t;$$

$$(24) \quad - \lambda_t\, \beta^t + \lambda_{t+1}\, \beta^{t+1} (K - B U_t) + \gamma_t\, \beta^t U_t F_t;$$

$$(25) \quad (\delta R_t/\delta M_t)\, \beta^t + \theta_{t+1}\, \beta^{t+1} (\delta F_t/\delta M_t) + \gamma_t\, \beta^t U_t (\delta F_t/\delta M_t) X_t;$$

$$(26) \quad - \theta_t\, \beta^t + \theta_{t+1}\, \beta^{t+1} (\delta F_t/\delta V_t) + \gamma_t\, \beta^t U_t (\delta F_t/\delta V_t) X_t; \text{ and}$$

$$(27) \quad (\delta \Omega/\delta X_T)\, \beta^T - \lambda_T\, \beta^T.$$

Setting (24) and (26) equal to zero gives the costate equations:

$$(28) \quad \lambda_t = \lambda_{t+1}\, \beta\, (K - B U_t) + \gamma_t U_t F_t, \text{ and}$$

$$(29) \quad \theta_t = \theta_{t+1}\, \beta\, (\delta F_t/\delta V_t) + \gamma_t U_t (\delta F_t/\delta V_t) X_t.$$

By setting (22) and (27) to zero, the value of $\gamma_t = \delta R_t/\delta H_t$ for t = 0, ..., T-1 and $\gamma_T = \delta \Omega/\delta V_T$ for T. In addition, as the shadow value of management and timber growth are zero at time T, $\theta_T = 0$. These values can be

substituted into (28) and (29) as required.

4.2 Methods for Solving the Dynamic Problem

Burt and Cummings (1977, pp. 16-17) discuss several important points about the use of dynamic models. Firstly, discrete models are important in natural resource economics since the decision mechanism in resource management will of necessity be applied discontinuously with respect to time. Secondly, specification of the decision variables and the appropriate state variables is an important part of model formulation. That is, model specification is important. Finally, the equations of motion for natural resources are inherently stochastic. Although we specify a deterministic model in this section, it is necessary to specify a stochastic one if uncertainty is included in the model.

There are a number of different methods for solving the discrete-time, dynamic problem (8) or (20). Some of these methods, including the one employed in this study, are considered below.

4.2.1 Gradient Method

One approach to solving the foregoing problem is to employ the gradient technique (Burt and Cummings 1970; Lyon and Sedjo 1983). The Lagrange multipliers λ_t and θ_t are the gradient vectors of U_t and M_t, respectively. That is, λ_t and θ_t are the derivatives of the objective function with respect to the controls U_t and M_t, respectively, at each point in time. We wish to derive values for the multipliers λ_t and θ_t through an iterative process. Consider first λ_t. We employ the first-order condition for the state variable (X_t) associated with this co-state variable, namely, equation (28).

It is possible to determine the value of λ_T from (27) and values of λ_{T-1}, λ_{T-2}, and so on from (28). This gives the following:

$$\lambda_T = \delta\Omega/\delta X_T$$

$$\lambda_{T-1} = \gamma_{T-1}\, U_{T-1}\, F_{T-1} + \lambda_T\, \beta\, (K - B\, U_{T-1})$$

$$= (\delta R_{T-1}/\delta X_{T-1})\, U_{T-1}\, F_{T-1} + \delta\Omega/\delta X_T\, \beta\, (K - B\, U_{T-1})$$

$$\lambda_{T-2} = (\delta R_{T-2}/\delta X_{T-2})\, U_{T-2}\, F_{T-2}$$

$$+ \beta\, (K - B\, U_{T-2})[(\delta R_{T-1}/\delta X_{T-1})\, U_{T-1}\, F_{T-1} + \delta\Omega/\delta X_T\, \beta\, (K - B\, U_{T-1})]$$

.....

In general,

$$(30)\quad \lambda_t = \delta R_t/\delta X_t\, U_t\, F_t \sum_{k=t+1}^{T-1} (\delta R_k/\delta X_k)\, U_k\, F_k \prod_{j=t}^{k-1} \beta\, (K - B\, U_j)'$$

$$+ \delta\Omega/\delta X_T \prod_{j=t}^{T-1} \beta\, (K - B\, U_j)'.$$

Similarly, we use the first-order condition for the state equation for V_t associated with the co-state variable θ_t, namely, Equation (26), to derive the following:[9]

$$(31)\quad \theta_t = [(\delta R_t/\delta H_t)\, U_t\, X_t$$

$$+ \beta \sum_{k=t+1}^{T-1} (\delta R_k/\delta H_k)\, U_k\, X_k\, (\delta F_k/\delta V_k) \prod_{j=t}^{k-1} \beta\, (\delta F_j/\delta V_j)]\, (\delta F_k/\delta V_k)$$

Under the gradient method, an initial set of controls is assumed for the entire time horizon. The procedure begins in period T and works its way to the present. Consider the controls U and Lagrangians λ and θ. In period T, we know the value of $\lambda_T = \delta\Omega/\delta X_T$ by setting (27) to zero, while $\theta_T = 0$ since

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[9]Note that $\theta_T = 0$ since management at time T has no effect on the objective function. Further, the same relationships could have been obtained for the earlier problem (8), although the expressions would be more complicated as another counter would be involved.

management at time T has no effect on the objective function. We also know (i) that $\gamma_t = \delta R_t/\delta H_t$ for t = 0, ..., T-1; (ii) what the objective functional R_t is for all t; and (iii) the values of K and B. Finally, we must choose an initial set of controls (U_t and M_t) for all t; these will subsequently be modified as part of the optimizing algorithm.

Whenever the value of the Lagrangian is positive, this implies that the objective function is increased when the associated control is increased. Likewise, a negative value of the Lagrangian indicates that the associated control for that period should be reduced, while a Lagrangian of zero implies that there should be no change in the associated control for that period. Using the gradient method, the controls are increased, reduced or unchanged by a pre-specified amount for each iteration. Thus, the objective function is maximized by this "hill-climbing" approach. An example of the approach, but with no discussion of the mechanism, is provided by Lyon and Sedjo (1983).

4.2.2 Dynamic Programming

Dynamic programming (DP) offers an approach to solving the discrete time optimal, forest management problem which is rather straight-forward. The DP solution requires that both the state and the control variables be discrete or that intervals for these variables be constructed--that is, neither the state nor the control variables can be continuous.[10] If the states and controls take on too many values (or the intervals are too narrow so that there are too many) or there are too many state and control variables, then the so-called "curse of dimensionality" is encountered. Despite this,

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[10] In practice, of course, the same is true of the other methods.

however, for many practical problems DP is an excellent tool.[11] DP can be used both when the state variables in time t+1 depend upon the state and control variables at time t in a deterministic or stochastic fashion. In this section, we employ a deterministic model, while in the next section a stochastic DP model is used to examine forest management decisions in the context of uncertainty. The DP methodology is discussed by Bellman (1957), Howard (1960), Kilkki and Vaisanen (1969), Burt and Cummings (1970), Hastings (1973), Kennedy (1986), Buongiorno and Gilless (1987), and others.

Before discussing this method, however, it is useful to consider substituting (30) into (23) and (31) into (25). Upon doing this, we get relationships similar to Burt and Cummings' (1970, p. 581) relations (17) and (18). These indicate that the dependence structure in the decision process is Markovian, which then permits the use of DP.

The discrete dynamic optimal control problem (8) or (20) takes into account the fact that decisions (i.e., the choice of control levels) in period t affect the values of the state variables in period t+1 and, hence, the returns that can be earned in future periods--these are appropriately discounted by β. This is unlike single period or static optimization. The principle governing the DP method is Bellman's Principle of Optimality: "An optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman 1957, p. 83).

The DP method is as follows. Beginning in period T, we calculate the value of the salvage function for all combinations of the state variables.

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[11] See Kennedy (1986) for examples related to agriculture and natural resources.

In period T-1, for every possible set of controls and state variables, the current value of the benefit function, plus the value of the salvage under the various sets of associated controls, is calculated. The controls which maximize this sum for each possible combination of state variables in period T-1 are chosen. Then the procedure is repeated for period T-2, T-3, and so on. Upon reaching the current period, the values of the state variables are observed and the process is reversed to obtain the optimal path.

For problem (20) the recursive dynamic programming equation is:

$$(32) \qquad Z_t(X_t, V_t) = \max_{U_t, M_t} [R_t(H_t, U_t, M_t) + \beta Z_{t+1}(X_{t+1}, V_{t+1})].$$

Assuming differentiability of all the necessary functions and that an interior solution holds, necessary conditions for a maximum are:

$$(33) \quad (\delta R_t/\delta U_t) + (\delta R_t/\delta H_t)(\delta H_t/\delta U^t) + \beta (\delta Z_{t+1}/\delta X_{t+1})(\delta X_{t+1}/\delta U_t) = 0 \quad \text{or}$$

$$(\delta R_t/\delta U_t) = \beta \lambda_{t+1} B X_t - (\delta R_t/\delta H^t) F_t X_t.$$

$$(34) \quad (\delta R_t/\delta M_t) + (\delta R_t/\delta H_t)(\delta H_t/\delta U^t) + \beta (\delta Z_{t+1}/\delta V_{t+1})(\delta V_{t+1}/\delta M_t) = 0 \quad \text{or}$$

$$(\delta R_t/\delta M_t) = -\beta \Theta_{t+1} (\delta F_t/\delta M_t) - U_t (\delta F_t/\delta M_t) X_t.$$

These results are identical to those found in Equations (23) and (25), respectively.

If the forest land and the stock of trees are optimally managed across all periods, the resource owner is indifferent between leaving the acre of land in forest (letting it grow) or cutting the trees. This is seen by differentiating both sides of (32) by X_t and V_t, respectively.

$$(35) \qquad \delta Z_t/\delta X_t = (\delta R_t/\delta H_t)(\delta H_t/\delta X^t) + \beta (\delta Z_{t+1}/\delta X_{t+1})(\delta X_{t+1}/\delta X_t)$$

$$\Rightarrow \qquad \lambda_t = (\delta R_t/\delta H_t)(\delta H_t/\delta X^t) + \beta \lambda_{t+1} (K - B U_t).$$

$$(36) \qquad \delta Z_t/\delta V_t = (\delta R_t/\delta H_t)(\delta H_t/\delta V^t) + \beta (\delta Z_{t+1}/\delta V_{t+1})(\delta V_{t+1}/\delta V_t)$$

$$\Rightarrow \quad \Theta_t = (\delta R_t/\delta H_t)(\delta H_t/\delta V^t) + \beta \Theta_{t+1} (\delta F_t/\delta V_t).$$

The results in (35) and (36) are identical to (28) and (29), respectively.

Therefore, dynamic programming gives the same results as the Lagrangian solution to the discrete time, optimal control problem. It can also be demonstrated that it gives the same solution as the maximum principle (Kennedy 1986, pp.20-23).

If the objective or returns function changes over stages, or if the other functional relationships vary between states, the optimal strategy is found using the recursive relationship (32). Non-stationarity of the objective function requires that a value-iterative procedure be employed (see Hastings 1973; Kennedy 1986). Beginning at the end of the time horizon, this procedure determines the optimal path for each possible state. Once the current period is reached and the current state variable(s) is known, then the decision is made--the control is applied--and the optimal path is followed to the end of the time horizon.

In this case, the optimal paths of the state and control variables are found as follows. First, the optimal decision for each of the state variables, X_T and V_T, is made at time T--the last period in the planning horizon. This decision will generally be to clearcut all the forest so long as the benefits of cutting exceed the costs, which will be the case unless trees are simply too small. The salvage values can also be used as these will depend upon the age and size distribution of the trees left standing at the end of the planning period. Whatever the case, in time period T-1, the values of Z_T are known. The optimal values of $Z_{T-1}(X_{T-1}, X_{T-1})$ are then found for each combination of the state variables and decisions (U and M) using the recursive equation (32). The recursive process is then continued to the present time period. At that time, the actual values of the state variables, X_0 and V_0, are known and the process is reversed, thereby finding

the optimal path of both the state and the control (decision) variables.

The expected return for a given value of the state variable under the value-iterative procedure is simply equal to the value of the objective function for the period in question. This is also the long-run expected return if the salvage value--the value of the land at time T--is included as part of the objective function.[12]

The recursive relationship described above converges to a constant strategy if the time horizon is chosen sufficiently long (T is large) and returns in each state are constant in every time period--that is, the objective function is stationary. In addition, the equations of motion must be unchanging over time. The optimal policy can be found by the policy-iteration approach (Howard 1960; Hastings 1973). The method is to carry out the iterations of the recursive relationship (32) until a number of repetitions of the same policy occur. This policy is checked to determine if it is optimal. If it is not optimal, additional iterations are performed until a new strategy is found. The new strategy is then checked for optimality and the process is repeated until a constant solution is found.

In order to get a stationary strategy, it is necessary to assume that the benefit function R(H, V, M) is not explicitly a function of time (Burt and Cummings 1970, p. 588). Then the dynamic programming problem can be reduced to a single recursive equation which needs to be solved for period n, where n is the number of years left in the planning horizon. The solution algorithm begins at the end of the multi-stage process in much the same way as described above and as in the gradient method.

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[12]The salvage value should not be confused with soil expectation. Salvage value includes soil expectation plus the value of the standing timber which remains on the site.

The recursive relationship is as follows:

$$Z_0(X) = \Omega(X)$$

$$Z_1(X, V) = \max_{U,M} [R(H, V, M) + \beta Z_0(X)]$$

$$Z_2(X, V) = \max_{U,M} [R(H, V, M) + \beta Z_1[(KX + BUX), F(V,M)]$$

.....

$$Z_n(X, V) = \max_{U,M} [R(H, V, M) + \beta Z_{n-1}[(KX+BUX), F(V,M)].$$

Given that time is infinite, convergence to a constant strategy will occur as long as the benefit function is time invariant.

The dynamic programming method is employed in this study. In section 4.3, we extend the deterministic DP methodology to that of stochastic dynamic programming. Stochastic DP is required to deal with uncertainty in timber growth due to forest fire, insect pests and so on. Examples of both the deterministic and stochastic DP methods are provided in section 5. However, before proceeding to discuss this method, it is beneficial to point out that it may be possible to find an approximately optimal decision rule in cases where a steady state solution occurs. This is done in the next section.

4.2.3 Approximation Method

Burt and Cummings (1970, 1977) suggest the use of an approximation procedure to solve discrete time, optimal control problems. The advantage of this method is that it is simpler than either the DP or gradient method, but it requires that the real world falls into the region of approximation. In addition, the approximation method can be used only if the returns function in the objective does not change from one period to the next and that the

equations of motion (1), (2) and (3), plus the harvest function (4), remain invariant over the time horizon. Thus, a steady state solution can be identified. The method is illustrated below.

For the steady state solution, it is not necessary to keep time subscripts on many of the variables. From equation (33), recognizing that $\beta=1/(1+r)$, we obtain:

(37) $\lambda_{t+1}/(1+r) = [\delta R/\delta U + (\delta R/\delta H)\ (\delta H/\delta U)]/(B\ X)$.

From equation (35), and adding and subtracting λ_{t+1}, we obtain:

(38) $(\delta R/\delta H)\ (\delta H/\delta X) = [\lambda_{t+1}/(1+r)]\ [1 + r - K + B\ U] - (\lambda_{t+1} - \lambda_t)$.

Upon substituting (37) into (38) and rearranging, the equation for finding the optimal steady state control associated with each state is:

(39) $\delta R/\delta U +(\delta R/\delta H)(\delta H/\delta U) = BX\ [(\delta R/\delta H)(\delta H/\delta X) + (\lambda_{t+1} - \lambda_t)]/[1+r-K+BU]$.

If there is an optimal steady state stock X^*, with an optimal associated control U^* then, if $X_t = X^*$ and U^* is implemented, $X_{t+1} = X^*$ and $\lambda_t = \lambda_{t+1} = \delta Z/\delta X$ evaluated at X^*. Equation (39) then becomes

(40) $\delta R/\delta U +(\delta R/\delta H)(\delta H/\delta U) = B\ X\ (\delta R/\delta H)\ (\delta H/\delta X)/[1 + r- K + B\ U]$.

The optimal rule X^* can be derived from (39), but this requires knowledge of λ_{t+1} and λ_t which are not observable. However, the approximately optimal solution $U^a(X_t)$ is a good approximation to $U^*(X_t)$ if $\lambda_{t+1} - \lambda_t$ is small relative to $(\delta R/\delta H)(\delta H/\delta X)$ evaluated at X_t.

Insight into this can be obtained by considering Figure 1 which illustrates the Z(X:V) schedule in Z,X space, that is, assuming V is held fixed. The slopes of Z upon implementing the optimal policy U^* are λ_{t+1} and λ_t for X_{t+1} and X_t, respectively. Clearly, the slopes λ_{t+1} and λ_t will be closer the closer X_{t+1} is to X_t or the more linear Z(X:V) is over the range in question. It is in these situations that the approximately optimal

solution U^a provides a reasonable alternative to the actual optimal solution U^*.

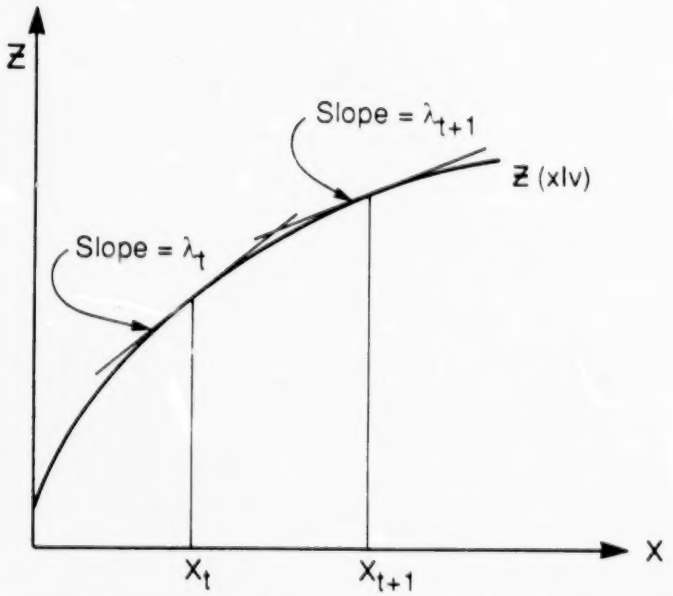

Figure 1: Illustration of the Approximately Optimal Solution

Likewise, the approximately optimal solution for the other control M_t can be found. From equation (34),

(41) $\quad \theta_{t+1}/(1+r) = U\,X + (\delta R/\delta M)/(\delta F/\delta M).$

From (36), and upon adding and subtracting θ_{t+1},

(42) $\quad (\delta R/\delta H)\,(\delta H/\delta V) = [\theta_{t+1}/(1+r)]\,[\delta F/\delta V + 1 + r] - (\theta_{t+1} - \theta_t).$

Substituting result (41) into (42) gives an expression similar to (38), namely,

(43) $(\delta R/\delta M)/(\delta F/\delta M) = -U\,X + [(\delta R/\delta H)\,(\delta H/\delta V) + (\theta_{t+1} - \theta_t)]/[\delta F/\delta V + 1 + r].$

Again, the optimal rule can be derived from (43), but the values of θ_{t+1} and θ_t are not known because they cannot be observed. Assuming that $\theta_{t+1} - \theta_t$ is small relative to $[(\delta R/\delta H)\ (\delta H/\delta V)]$ evaluated at V_t enables us to use an approximately optimal control $M^a(V_t)$ rather than the truly optimal control

$M^*(V_t)$.

Burt (1964, 1981) shows that, in practice, the approximately optimal decision rule is very close to the optimal decision. He demonstrates this for a groundwater problem (1964) and for soil conservation in the Palouse region of eastern Washington and western Idaho (1981).

4.3 A Stochastic Formulation of the Forest Management Problem

In this section, we consider the stochastic control problem. This problem is discussed in section 4.3.1. In section 4.3.2, we consider the use of stochastic dynamic programming to solve the stochastic control problem.

4.3.1 Stochastic Optimal Control Theory: A Brief Discussion[13]

The state transformation equation for a dynamic deterministic system may be described by the following difference equation:

(44) $\quad x(t+1) = f(t, x(t)),$

where $t \in T = \{0, 1, \ldots\}$. In order to incorporate uncertainty in this system, we assume that $x(t+1)$ is a random variable. Equation (44) then becomes:

(45) $\quad x(t+1) = f(t, x(t)) + v(t, x(t)), \quad t \in T,$

where f is the conditional mean of the random variable $x(t+1)$--conditioned on $x(t)$--and v is a random variable with mean zero and finite variance $\sigma^2(t, x)$. The conditional distribution of the random variable v, given $x(t)$, is independent of $x(s)$, $s < t$; that is, the process $\{x(t)\}$ is assumed to be Markovian. Now assume that the conditional distribution of v is normal and

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[13]The discussion in this section summarizes material found in Malliaris and Brock (1982).

define the random variable $u(t) = v(t)/\sigma(t,x)$. Then $u(t)$ is normally distributed with mean 0 and unit variance, and $u(t)$ is independent of x. Thus, $\{u(t)\}$ is a sequence of identical, independently distributed normally distributed random variables each with zero mean and unit variance. Equation (45) then becomes the stochastic state transformation equation

$$x(t+1) = f(t, x(t)) + \sigma(t,x(t))\, u(t), \quad t \in T. \tag{46}$$

The continuous time equivalent of the discrete state transformation equation (44) is $dx/dt = f(t, x(t))$, $t \in T = [o,\infty)$. The continuous time equivalent of the stochastic state transformation equation is

$$dx = f(t, x(t))\, dt + \sigma(t, x)\, dz, \tag{47}$$

where $\{z(t)\}$ is a Weiner or Brownian motion process with mean 0 and unit variance.

A Brownian motion or Weiner process $W(t)$ has the following properties:

(i) Sample paths of $W(t)$ are continuous.

(ii) $W(0) = 0$ by convention.

(iii) For $t, s \in T$, the increment $W(t+s) - W(s)$ is normally distributed with mean 0 and variance $\sigma^2 t$.

(iv) Given $t, s, q, r \in T$, then, if (t,s) and (q,r) are disjoint intervals, $W(s) - W(t)$ and $W(q) - W(r)$ are independent random variables.

Although these properties imply several important results, the main result of interest for our brief discussion is that $W(t)$, while continuous, cannot be differentiated. Sample paths are continuous in the sense that no jumps are permitted, but the paths can have sharp changes in direction (e.g., cusps) so that it is not possible to differentiate the sample path.[14]

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[14] See Malliaris and Brock (1982, pp.121-24) and Mangel (1985, pp.22-27) for a discussion of jump processes.

4.3.2 Stochastic Dynamic Programming

Use of stochastic dynamic programming has become increasingly common in recent years following the pioneering applications by Burt in the 1960s and early 1970s (Burt and Johnson 1967; Burt 1981). Recent applications include identification of optimal strategies for pest control (Zacharias, Liebman and Noel 1986; Zacharias and Grube 1986), rangeland management (Karp and Pope 1984), irrigation scheduling (McGuckin et al. 1987), and crop selection to avoid uncertain soil erosion (Weisensel 1988). Kennedy (1986, p.7) is surprised that most forestry problems have been formulated as deterministic rather than stochastic.

There are two ways in which uncertainty can be incorporated in the state transformation equations. (1) Uncertainty can be modeled in discrete time by assuming that the state transformation equation is of the form given by equations (45) or (46). In this case, it is necessary to first estimate the transformation equation. At the same time the stochastic process needs to be specified and the variance of the process determined. From the estimated equation, the conditional mean of x_{t+1}--the value of the state variable in time t+1--is found. Using knowledge of the density function of the stochastic process, it is possible to construct a probability transition matrix P for x_t:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ \cdots & & & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix} ,$$

where there are m possible levels of the state variable and the rows of P sum to 1.0. A different transition matrix exists for each value of the control

variable or decision taken.

Thus, a key ingredient in the stochastic dynamic programming problem is the state transition matrix P. The transition matrix gives the probabilities of attaining a particular level of the state variable (say volume of commercial timber growing on a site) in the next period, given (i) the current level of the state variable (volume of commercial timber) and (ii) the current value of the control variable (e.g., amount of effort expended on management techniques such as forest fire control). The probability is greater that the volume of timber available in the next period is higher when more funds are expended on forest management as opposed to less. Thus, the expected return in the next period will be higher if more effort is devoted to forest management in the current period. However, the net return during the current period will be lower (indeed negative) as more money is spent on management. The DP methodology takes this fact into account, as well as the need to discount future returns.

The dynamic programming problem can be reduced to a single recursive equation which needs to be solved for period n, where n is the number of periods left in the planning horizon. Mathematically, the standard problem is to choose a value of the control variable u (various levels of expenditures on forest management) which maximizes:

$$f_n(i) = \max_u \left[R^u(i) + \beta \sum_{j=1}^{m} P^u(i,j) f_{n-1}(j)\right], \tag{48}$$

where $f_n(i)$ is the discounted value of future returns, given the state variable is at level i at the beginning of an n-stage process; $f_{n-1}(j)$ is the discounted value of future net returns over the remaining n-1 periods of an n-period process, given state variable level is j and that the optimal path

is followed; $R^u(i)$ is the current net return for decision u,[15] given the state variable is at level i; $P^u(i,j)$ is the probability of moving from the current level i of the state variable to level j in the next period, given that the current decision is u (some level of expenditures on forest management); and $\beta = 1/(1+r)$, where r is the real rate of discount between periods.

The stochastic dynamic programming problem is recursive, and the solution algorithm begins at the end of the multi-stage process. Further, it is assumed that the value of the state variable in a given period has no influence on the level of the state variable two periods in the future; that is, the process is Markovian. Hence, Markov programming can be used to solve this problem (Hastings 1973).

The recursive relationship (48) converges to a constant strategy if the time horizon is chosen sufficiently long (n is large) and returns to forest management within states are constant in every time period--that is, the objective function is stationary. The optimal policy can be found by the algorithm employed by Burt and Allison, and initially described by Howard as the policy-iteration approach. Modifications of this algorithm are found in Hastings (1973) who also discusses optimal stopping rules. The method is to carry out the iterations of the recursive relationship (48) until a number of repetitions of the same policy occur. This policy is checked to determine if it is optimal. If it is not optimal, additional iterations are performed until a new strategy is found. The new strategy is then checked for

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[15] If decision maker risk is to be taken into account in the dynamic problem, then, with Katoaka's safety-first criterion, the current period returns are replaced by the current risk-adjusted returns. This is illustrated by equations (2), (3) and (5) in Pyle and Turnovsky (1970, p.76).

optimality and the process is repeated until a constant solution is found.

(2) In approach (1), it is assumed that the parameters in the state transformation or growth equation are known with certainty but that other factors are the cause of uncertainty. Alternatively, it is possible to specify uncertainty as a stochastic process affecting the parameters of the growth or transformation equation. In this case, $x_{t+1} = f(x_t, u, \theta)$, where u is the control or decision variable and θ is a parameter that follows a stochastic process. Pindyck (1980) specifies this process as

$$d\theta = \sigma(\theta)\, dz = \sigma(\theta)\, \varepsilon(t)\, \sqrt{dt}$$

for the continuous case, where z is a stochastic process and $\varepsilon(t)$ is a serially uncorrelated normal random variable with zero mean and unit variance--that is, dz is a Weiner process. As noted previously, Lewis (1981) used a similar method to specify uncertainty in the management of the fishery, but we will use the first approach of modeling uncertainty via the transition matrices.

5. THE PROJECTION OR LESLIE MATRIX

Kennedy indicates that it might be possible to include the projection or Leslie matrix (Leslie 1945, 1948) in the state transformation equation (1986, pp.268-69). The Leslie equation may permit the inclusion of stochastic events (e.g., stochastic price, yield), nonlinearities and so on (Pielou 1977; Pollard 1966; Usher 1966, 1969, 1972).

In the foregoing model, the state transformation equations can be written as a Leslie equation:

(49) $$x_{t+1} = L\, x_t - h_t,$$

where x_t is a vector representing the forest stock age distribution at time t, L is the Leslie matrix, and h_t is a vector of harvests of different age trees at time t. The projection or Leslie matrix is as follows:

$$L = \begin{bmatrix} F_0 & F_1 & \cdots & F_{k-1} & F_k \\ p_{01} & 0 & \cdots & 0 & 0 \\ 0 & p_{12} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & p_{k-1,k} & 0 \end{bmatrix}$$

The elements of the Leslie matrix are the fecundity parameters for each age group (F_i) located in the first row and the probabilities (p_{ij}) of surviving to the next age group which are located on the lower off-diagonal of the matrix. Since older cohorts may no longer be able to reproduce, elements in the far right-hand columns of the first row may be zero. In this case, the matrix needs to be modified but this makes no difference in the method of analysis (see Pielou 1977, pp.42-44).

A number of researchers have recommended the use of the Leslie matrix for studying population growth (Leslie 1945, 1948; Pielou 1977). However, they point out that one drawback of the projection matrix is that it assumes

fecundity and survival are independent of population density. Pielou (1977, pp.55-57) suggests a method for including dependence on population density, but this method appears less than satisfactory.

In forestry, Usher (1966, 1969, 1972) has used the Leslie approach, but his objective is simply to maximize the growth of a forest, and thereby maximize the annual harvest, although Mendelssohn (1976) argues that Usher's formulations are characterized "by a certain vagueness of purpose and problem" (p.341). Our objective, on the other hand, is to maximize an economic function such as net discounted revenues or some specification of social welfare. Pollard (1966) suggests ways for including stochastic parameters in the Leslie matrix. This can be done, for example, by making the p_{ij} in L stochastic. The Leslie matrix can also be extended to make forest growth nonlinear by adding a functional form within the matrix, and climate parameters affecting growth can also be added.

Mendelssohn (1976) criticizes many specifications employing the Leslie or projection matrix because their purpose is to maximize population growth rather than harvests. The economist's focus is not even on harvests per se, but, rather, on some value such as net returns which is related to harvest. In particular, the economist seeks to maximize the expected net present value of the resource which is managed. Further, Mendelssohn criticizes the Leslie model because "the intuitive feeling that over a long planning horizon an optimal harvesting policy will keep the population size in check is not true with the Leslie model" (p.349).

As a result of the foregoing discussion and and our view that, at this time, nothing will be gained by employing a Leslie model, we do not develop a stochastic model via the use of a projection or Leslie matrix.

6. AN EXAMPLE OF A STOCHASTIC DYNAMIC PROGRAMMING MODEL FOR FOREST MANAGEMENT

In this section, we provide an example stochastic DP model of forest management. In particular, we compare the results of the stochastic model with those of a deterministic one; that is, we compare optimal management strategies in a certain environment with those under uncertainty. Due to time limitations in the current research, the example is somewhat contrived. Although the model is simple and it is unlikely that a model which takes into account the fully complexity of the forestry management problem can be built, we offer the following defense by Walters (1986).

> "Management decision making and science are not the same thing... It is reductionist myth that models become monotonically better as the details of natural processes are more fully articulated. Practical problems of structural specification and estimation imply that there is some balance point or 'optimum complexity' beyond which model performance will actually deteriorate" (pp.185-86)

However, the model we construct can be extended by using data on actual forests in Alberta, Saskatchewan or elsewhere simply be estimating appropriate relationships as required below. Given the time constraint, we do not attempt to include risk at this time.

6.1 Introduction to the Example

The model developed in this section employs wood volume as the state variable and the objective is to maximize returns as a function of wood volume. The model is applicable to the production of pulp as, in this case, one is not concerned with trunk diameter but rather with wood volume. The problem is to find the optimal rotation length given that wood growth is (1) deterministic and (2) that it is uncertain or stochastic. Uncertainty in forest growth due to forest fires, disease or insect pests is modeled as a

random shift about the actual growth function according to a stochastic process. The unit of measurement is a square kilometre of forest.

The data for the model is based roughly on the Edson Forest of west-central Alberta. This forest is part of the Boreal forest of western Canada. The Edson forest has an area of 50,000 km^2 and an estimated wood volume of 210 million m^3 (Northern Forest Research Centre 1982). A forest growth equation or state transformation equation is postulated. The data used to estimate the wood volume growth is somewhat contrived, but the main purpose of this paper is to provide an example of deterministic versus stochastic dynamic programming applied to a forestry problem.

The objective is to maximize the discounted returns from forest management strategies using dynamic programming. No attempt is made to assess nonmarket amenity values of the forest, such as scenic values, recreational value and the benefits of the forest as a pollution sink; that is, multiple use of the forest is ignored. Wood volume returns are the sole criterion for valuing the forest. In short, the economic problem is the timing of a sequence of harvests on a stand which is managed into the infinite future with prices, costs and stand productivity assumed to be known and unchanging. Further, it is assumed that the stand is of even age. Interdependence of stands is ignored so that management of one stand does not affect the value of another stand.

Two cases are investigated. (1) In the deterministic case, it is assumed that one knows with certainty how the state variable changes over time. (2) In the stochastic or uncertainty case, the change in the state variable (wood volume) is stochastic so that it is not possible to predict wood volume growth exactly over time. This case is a better representation

of reality since forest modeling involves uncertain events such as pests and forest fires which affect timber growth.

The problem of maximizing the net present value of returns from harvesting requires some knowledge of the biological growth rate of the forest under various management strategies. In this model, the control variables are three levels of management and whether or not to clearcut the stand. It is assumed that the management strategies are positively correlated with tree growth. In other words, intense management of the forest results in increased tree growth and, hence, faster growth of wood volume. This is assumed despite evidence indicating that some management strategies may be detrimental to tree growth in the long run.

6.2 Structure of the Model

6.2.1 Wood Volume Growth

The state transformation equation represents the growth of wood volume or biomass from one period to the next; it represents the equation of motion for the dynamic programming problem. It is assumed that wood volume in period t+1 depends only on the volume available in period t and the management strategy followed at time t. Thus, the process is Markovian.

Biomass or timber volume is assumed to follow a logistics growth function (Hartwick and Olwiler 1986; Clark 1976). No attempt was made to test the correctness of this functional form. The logistics growth function is chosen for its simplicity, although it is a feasible candidate to represent timber biomass growth as this functional form does embody many reasonable assumptions with regard to population or biomass growth. For example, Hartwick and Olewiler (1986, pp.352-54) use the logistics growth

function to explain forest rotations. They state that a forest stand with trees of identical age could be characterized by the logistics growth function.

The strongest assumption of logistics growth is that the rate of growth of the stock (wood volume) depends on the amount of stock. This seems to be a reasonable assumption for a stand of trees. With a small amount of biomass, growth rates are faster than with large amounts of biomass since there is less competition for nutrients, water and sunlight. However, as biomass increases, it grows at a slower rate because the denser growth encourages spread of disease and leads to increased competition. Eventually, death and decay will equal regeneration, at which point the biomass does not increase or decrease. This point is referred to as the carrying capacity (point k in Figure 2). Biomass increases at a decreasing rate, achieving a maximum at k/2, which is maximum sustainable yield harvest. Growth of wood volume in the stand continues until k, the carrying capacity.

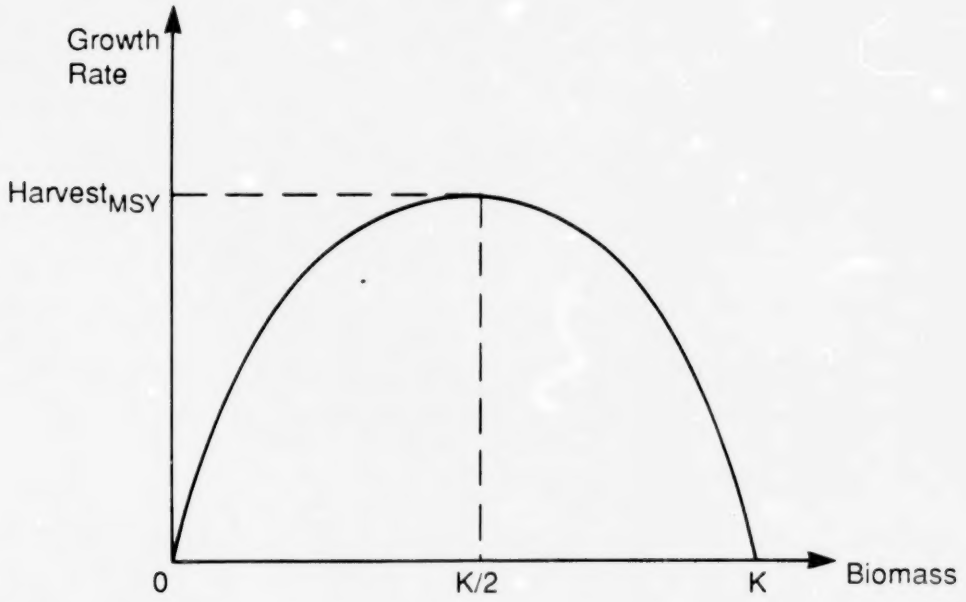

Figure 2: The Logistics Growth Function

The mathematical form of the logistics growth function is as follows:

$$dx/dt = rx(1-x/k) \qquad \text{in continuous time} \qquad (6.1)$$

$$x_{t+1} - x_t = rx_t(1-x_t/k) \qquad \text{in discrete time} \qquad (6.2)$$

where x_t refers to the biomass at time t, r is the intrinsic growth rate, and k is carrying capacity. In this study, the discrete version of the logistics growth function (6.2) is used. It is assumed that the length of time between stages is three years; that is, the length of time between t and t+1 in (6.2) is assumed to be 3 years.

It is necessary to obtain estimates of the carrying capacity (k) and the intrinsic growth rate (r). The carrying capacity in this model is the maximum amount of biomass that a square kilometre of land can support. The intrinsic growth rate refers to the rate of growth when the amount of biomass is close to zero. In this model, the carrying capacity is assumed to be fixed, but it could vary depending on stochastic variables such as an epidemic disease and environmental factors. Futhermore, clear cutting may alter the ecosystem to an extent where the type and amount of vegetation that the land can support is permanently changed. In particular, soil erosion due to clear cutting may permanently alter the carrying capacity of the land.

The intrinsic growth rate was chosen arbitrarily due to lack of data and was varied according to the intensity of management. Increased management was assumed to increase the intrinsic growth rate in a positive fashion. The intrinsic growth rates assumed in this application are:

with no management, $r = 0.15$;

with a management represented by expenditures of \$400/km^2, $r = 0.30$; and

with a management represented by expenditures of \$800/km^2, $r = 0.45$.

The management expenditures are over a period of three years. Using the

annual allowable cut figure for the Edson forest and its current volume of 4,200 m^3/km^2, the carrying capacity was estimated to be 4,410 m^3/km^2.[16] With this information, four equations of motion corresponding to the three levels of management and clear cutting are constructed using the logistics form (6.2).

$x_{t+1} = .15x_t(7.66-x_t/4410)$ with no management costs

$x_{t+1} = .30x_t(4.33-x_t/4410)$ with \$400/km^2/3yr management costs

$x_{t+1} = .45x_t(3.22-x_t/4410)$ with \$800/km^2/3yr management costs

$x_{t+1} = 0$ after clear cutting

In the dynamic programming problem, the equations of motion are embedded in the transition matrix P, which is discussed below.

6.2.2 State Variable Ranges, Prices and Discount Rate

Since the DP problem is discrete, it is necessary to divide biomass volume, which ranges from 0 m^3/km^2 to 4,410 m^3/km^2, into discrete categories. The midpoints for each category are then used in the calculations. The separation of wood volume into categories, which are referred to as states, cannot be arbitrary as the divisions must be fine enough to reflect growth from one stage to the next. Further, the divisions must be sufficiently fine to reflect variation in growth from various management strategies. If variation in strategies is not reflected, the problem becomes trivial and the results useless. In this model, forty five categories or states are used. It should be noted that it is better to use as few states as possible because, as the number of states increases, the matrices become very large

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[16] Information for generating this data is available in Northern Forest Research Centre (1982).

and the curse of dimensionality may be encountered.

An arbitrary price of $\$36.60/m^3$ was used, but it already has all expenses deducted except the management costs. This implies that a constant cost of production is assumed. However, to make the problem more realistic, it was necessary to put a restriction on cutting in the lower states to reflect higher costs of cutting small trees.

Finally, a social discount rate is used as opposed to a private discount rate because the forest is ultimately owned by the public. The social discount rate reflects the value of the forest to society and is usually lower than the private discount rate or interest rate private firms face when borrowing money. The guidelines for choosing a societal discount rate are hazy, but we decided to chose a rate of 2.5 percent per annum. The annual rate was then converted to a three-year rate in the computer program.

6.2.3 Formulation of the Problem

The forestry management problem is as follows:

$$\max_{m} \left[36.6\, x_{t^*}\beta^{t^*} + 36.6\, x_{2t^*}\beta^{2t^*} + \ldots + 36.6\, x_{nt^*}\beta^{nt^*} - \sum_{t=1}^{T} C_t(m_t)\, \beta^t\right]$$

s.t.
$$x_{t+1} = .15x_t(7.66 - x_t/4410) \quad \text{with no management costs}$$
$$x_{t+1} = .30x_t(4.33 - x_t/4410) \quad \text{with } \$400/km^2/3yr \text{ management costs}$$
$$x_{t+1} = .45x_t(3.22 - x_t/4410) \quad \text{with } \$800/km^2/3yr \text{ management costs}$$
$$x_{t+1} = 0 \quad \text{after clear cutting}$$

where price is $\$36.6/m^3$, β is the discount rate, x_t is the volume of wood at time t, t^* refers to the optimal rotation length, $C_t(m_t)$ are management costs per 3-year period, n is the number of rotations in the planning horizon, and T is the number of periods in planning horizon. The objective

is to maximize returns from wood volume subject to the three management strategies (m) or clear cutting. Returns accrue once every rotation period, after cutting the trees. The returns from the number of rotations in the planning horizon are discounted by the social discount rate. Management costs may be incurred in every stage or period, and are also discounted at the social rate. The growth of wood volume is governed by the equations of motion for each management strategy. Finally, whatever happens after time T is of no concern and has no value.

6.2.4 The Transition Matrices

Two transition matrices are constructed, one for each of the deterministic and stochastic problems.[17] Since 45 discrete categories for wood volume are employed, the state/decision transition matrix has dimensions 180 X 45. There are 180 rows since there are 45 states and 4 decisions for each of the states. Thus, every subset of 4 rows represents one state and each row in the subset represents one of the four possible actions. Each of the columns represents the wood or biomass volume in the next period (stage) depending upon which decision was taken. For the deterministic model, 1s and 0s are used to embed the equations of motion in the matrix. The 1s indicate the movement from the present period state to the next period state under a given management strategy and 0s indicate that these states are unattainable from the original state and decision. Thus, in each row a single 1 should be present since the probability distribution is a spike; this is the case of no uncertainty. As the dimensions of the matrix are large, only some of the

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[17] It is not necessary to construct a transition matrix for the deterministic problem and, indeed, it adds to the complexity of the problem, but we do so for illustrative purposes.

entries of the deterministic transition matrix are presented in Table A.1 of the Appendix.

When biomass growth is assumed to be stochastic--the uncertain case--, the probability transition matrix must be constructed. It is similar to the transition matrix discussed for the deterministic case, except that the probabilities are not spikes. In this model, it is assumed that variance increases as biomass increases because there is a greater chance of disease and fire when there is more biomass. The basic problem is to develop a probability distribution for the management strategy in each state. We developed an artificial probability distribution using the variances for the various states as presented in Table A.3. As noted, the probability of forest fires is also assumed to increase with biomass. A forest fire at time t is assumed to put the system into the first state in period t+1; hence, the probability of a forest fire is found in the first column in the transition matrix. It is assumed that a fire is similar in effect to clear cutting except no revenue is earned; there is no salvage value of wood after a fire. Forest fire probability increases with biomass from 0.01 to 0.03 in intervals of 0.01 every fifteen states. Once again, the entire transition matrix is too large to present in a single table; therefore, only some of the elements are provided in Table A.2 in the Appendix. The rows in the transition matrix must sum to 1.0.

6.3 Empirical Results

The fundamental equation of dynamic programming is used to solve the forestry management problem outlined in the preceding section. Technically, the model developed is solved as an infinite Markovian problem. To develop

the program to solve the optimal rotation, a computer program written in Turbo PASCAL was developed and run on a micro-computer.[18]

The output file for the computer program is extensive and for this reason has not been included. The program was run for a time horizon of one hundred periods (300 years). Before the rotation length can be determined, a stationary or steady state solution must be found to the infinite time horizon Markov problem. Once the optimal policy is determined, it remains the same for each state regardless of the period one considers. For this example, the criterion for finding the stationary policy is to compare the present value of net returns between successive periods. When one fails to find a difference in such returns, the problem has converged to a stationary policy.

The optimal policies for both the deterministic and stochastic problems are indicated in Table 6.1. The optimal rotation is determined, in a rather crude fashion, by examining the deterministic transition matrix and finding the number of years until clear cutting.[19] The optimal rotation turns out to be 98 years in the deterministic case and 74 years in the stochastic case. In the deterministic case, stands would be cut once they reached a biomass of 3,650 m^3/km^2. Stands that exceeded this state should be cut immediately, with the optimal strategy followed thereafter. Stands with a biomass less than 3,650 m^3/km^2 would be managed according to the optimal strategy. Under uncertainty, the rotation length is reduced as one would expect. The

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[18]The computer program is a modification of a program written by the author in conjunction with another problem.

[19]The proper way to determine the optimal rotation in the stochastic case is to employ Monte Carlo simulation over the state transformation (growth) equations.

harvesting of the forest would take place once biomass exceeds 2,950 m^3/km^2. Stands greater than this amount would be cut at once, while those less than this amount would follow the optimal management strategy indicated in Table 6.1.

This simple example provides evidence in support of the literature reviewed in section 3.2, namely, that a resource exploited quicker when uncertainty is present. In this example, the shorter rotations obtained with a stochastic model can be explained by the reduction in expected net present value. The expected value in the deterministic case is higher since there is no variance as the probability density function degenerates to a spike.

Table 6.1: Convergence to Stationary Policy in Deterministic Model

state	Wood Volume Mid-point (m^3)	Deterministic Model		Stochastic Model	
		Optimal Policy	Present Value of Return	Optimal Policy	Present Value of Return
1	10	1	0.00	1	0.00
2	100	1	0.00	1	0.00
3	250	2	55,978.09	2	46,010.63
4	350	2	60,713.03	2	50,432.23
5	450	1	65,812.05	3	55,494.67
6	550	3	70,872.38	3	58,069.93
7	650	1	71,672.38	2	62,093.78
8	750	3	77,183.32	3	64,638.42
9	850	2	77,583.32	3	68,242.16
10	950	3	79,362.43	3	69,238.23
11	1050	3	83,979.50	3	72,769.52
12	1150	2	84,379.50	3	75,405.34
13	1250	3	86,326.17	2	76,013.22
14	1350	3	88,771.15	3	77,892.51
15	1450	3	91,298.25	3	79,771.48
16	1550	2	91,698.25	3	83,750.35
17	1650	3	93,825.36	3	81,487.53
18	1750	3	96,458.33	3	85,946.52
19	1850	3	99,179.75	3	87,668.25
20	1950	3	99,179.75	3	89,343.73
21	2050	2	99,579.75	3	91,960.32
22	2150	3	101,901.16	3	94,842.56
23	2250	3	104,736.58	3	93,875.73
24	2350	3	107,667.25	3	96,286.68
25	2450	3	107,667.25	3	97,812.99
26	2550	2	108,067.25	3	100,821.91
27	2650	3	110,597.92	3	104,117.06
28	2750	3	113,651.36	3	107,447.76
29	2850	3	116,807.37	3	106,224.24
30	2950	3	116,807.37	4	107,970.00
31	3050	2	117,207.37	4	111,630.00
32	3150	3	119,963.37	4	115,290.00
33	3250	3	123,251.60	4	118,950.00
34	3350	3	126,650.27	4	122,610.00
35	3450	3	126,650.27	4	126,270.00
36	3550	3	130,048.94	4	129,930.00
37	3650	4	133,590.00	4	133,590.00
38	3750	4	137,250.00	4	137,250.00
39	3850	4	140,910.00	4	140,910.00
40	3950	4	144,570.00	4	144,570.00
41	4050	4	148,230.00	4	148,230.00
42	4150	4	151,890.00	4	151,890.00
43	4250	4	155,550.00	4	155,550.00
44	4350	4	159,210.00	4	159,210.00
45	4410	4	161,406.00	4	161,406.00

1 = no management; 2 = \$400/km^2; 3 = \$800/km^2; 4 = clear cut

7. CONCLUSIONS

In this study, the literature on risk and uncertainty as applied to renewable resource problems was reviewed and the importance of taking risk and uncertainty into account in forestry management demonstrated. In addition, powerful theoretical tools for analysis of inherently dynamic forestry management decisions were examined. The focus of the discussion was on risk and uncertainty and their inclusion in the dynamic decision models. Risk was defined as the risk attitude, generally risk averseness, of the decision maker, while uncertainty was defined as stochastic forest growth--that is, forest growth which is affected by uncertain events such as insect pest infestation, disease and forest fire. As there remains some controversy about how one should include risk into dynamic models, the focus in this particular study was on uncertainty regarding forest growth from one period to the next.[20] The current study recommends that stochastic dynamic programming be used to examine uncertainty in forest management. The usefulness of stochastic dynamic programming to examine issues of uncertainty in forestry management was demonstrated with a simple example. However, lack of data detracts from the example's usefulness at present. Modifications to the stochastic DP model can obviously be made to make the model more realistic. For empirical analysis, the model can be extended in several directions besides simply obtaining data appropriate to the region one wishes to study. It is possible to introduce replanting strategies after

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[20] As noted in the text, Van Kooten and Young (1988) employ a safety-first method for including risk in dynamic decision models. However, this paper has been criticized at the recent S-180 risk meetings in Savannah, Georgia, held on March 21-23, 1988. Other methods for including risk in DP models were also criticized at these meetings and, at this time, the methodological issues remain unresolved.

clear cutting and fires. Thinning costs, chemical spraying and specific expenditures on forest fire prevention are management strategies that can readily be introduced into the model, provided there is sufficient data available to estimate the impact of such activities. The increasing probability of forest fire and its destructive power could possibly be handled by introducing another state variable, forest debris, and the associated state transformation equation; as debris accumulates the chance and magnitude of the fire would increase. Thus, it is possible to allow for possible benefits that fires provide and it may be possible to include a salvage value after a fire, depending upon its destructive power. While many extensions to the simple problem are possible, new state variables increase the matrix size and make the problem difficult to solve--the curse of dimensionality is encountered. However, a similar problem occurs with any numerical solution of dynamic models, for example, numerical solution of a system of differential equations.

Finally, it is important to obtain input from forest managers in the models examined in this study. While the models are limited in terms of their complexity, forest managers and other researchers will be able to fine tune the components of the model so that the results are a better approximation to reality. However, the first step in this direction is the construction of a stochastic DP model based on actual field data for a particular forest or forest stand.

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APPENDIX

DATA TABLES

APPENDIX

DATA TABLES

Table A.1: The Transition Matrix for the Deterministic Model

		1	2	3	4	5	6	7	8		45
1	a	1	0	0	0	0	0	0	0	....	0
	b	1	0	0	0	0	0	0	0	....	0
	c	1	0	0	0	0	0	0	0	....	0
	d	1	0	0	0	0	0	0	0	....	0
2	a	0	1	0	0	0	0	0	0	....	0
	b	0	1	0	0	0	0	0	0	....	0
	c	0	1	0	0	0	0	0	0	....	0
	d	1	0	0	0	0	0	0	0	....	0
3	a	0	0	1	0	0	0	0	0	....	0
	b	0	0	0	1	0	0	0	0	....	0
	c	0	0	0	1	0	0	0	0	....	0
	d	1	0	0	0	0	0	0	0	....	0
4	a	0	0	0	1	0	0	0	0	....	0
	b	0	0	0	0	1	0	0	0	....	0
	c	0	0	0	0	1	0	0	0	....	0
	d	1	0	0	0	0	0	0	0	....	0
5	a	0	0	0	0	0	1	0	0	....	0
	b	0	0	0	0	0	1	0	0	....	0
	c	0	0	0	0	0	0	1	0	....	0
	d	1	0	0	0	0	0	0	0	....	0
	....			....			....			....	
42	a	0	0	0	0	0	0	....	1	0	0	0
	b	0	0	0	0	0	0	....	0	1	0	0
	c	0	0	0	0	0	0	....	0	1	0	0
	d	1	0	0	0	0	0	....	0	0	0	0
43	a	0	0	0	0	0	0	....	0	1	0	0
	b	0	0	0	0	0	0	....	0	1	0	0
	c	0	0	0	0	0	0	....	0	0	1	0
	d	1	0	0	0	0	0	....	0	0	0	0
44	a	0	0	0	0	0	0	....	0	0	1	0
	b	0	0	0	0	0	0	....	0	0	1	0
	c	0	0	0	0	0	0	....	0	0	1	0
	d	1	0	0	0	0	0	....	0	0	0	0
45	a	0	0	0	0	0	0	....	0	0	0	1
	b	0	0	0	0	0	0	....	0	0	0	1
	c	0	0	0	0	0	0	....	0	0	0	1
	d	1	0	0	0	0	0	....	0	0	0	0

a = no management; b = $400/sq km; c = $800/sq km; d = clearcut

Table A.2: The Transition Matrix for the Stochastic Model

		1	2	3	4	5	6	7	8		45
1	a	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
	b	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
	c	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
	d	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
2	a	0.01	0.99	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
	b	0.01	0.99	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
	c	0.01	0.99	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
	d	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
3	a	0.01	0.05	0.89	0.05	0.00	0.00	0.00	0.00	....	0.00
	b	0.01	0.00	0.08	0.83	0.08	0.00	0.00	0.00	....	0.00
	c	0.01	0.01	0.11	0.75	0.11	0.01	0.00	0.00	....	0.00
	d	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
4	a	0.01	0.00	0.05	0.89	0.05	0.00	0.00	0.00	....	0.00
	b	0.01	0.00	0.00	0.08	0.83	0.08	0.00	0.00	....	0.00
	c	0.01	0.00	0.01	0.11	0.75	0.11	0.01	0.00	....	0.00
	d	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
5	a	0.01	0.00	0.00	0.00	0.05	0.89	0.05	0.00	....	0.00
	b	0.01	0.00	0.00	0.00	0.08	0.83	0.08	0.00	....	0.00
	c	0.01	0.00	0.00	0.00	0.01	0.11	0.75	0.11	....	0.00
	d	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
6	a	0.01	0.00	0.00	0.00	0.00	0.05	0.89	0.05	....	0.00
	b	0.01	0.00	0.00	0.00	0.00	0.08	0.83	0.08	....	0.00
	c	0.01	0.00	0.00	0.00	0.00	0.01	0.11	0.75	....	0.00
	d	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	....	0.00
		....		....		....				....	
41	a	0.03	0.00	....	0.00	0.08	0.76	0.08	0.00	0.00	0.00
	b	0.03	0.00	....	0.00	0.03	0.10	0.71	0.10	0.03	0.00
	c	0.03	0.00	....	0.00	0.03	0.12	0.67	0.12	0.03	0.00
	d	1.00	0.00	....	0.00	0.00	0.00	0.00	0.00	0.00	0.00
42	a	0.03	0.00	....	0.00	0.01	0.08	0.79	0.08	0.01	0.00
	b	0.03	0.00	....	0.00	0.00	0.02	0.10	0.73	0.10	0.02
	c	0.03	0.00	....	0.00	0.00	0.03	0.12	0.67	0.12	0.03
	d	1.00	0.00	....	0.00	0.00	0.00	0.00	0.00	0.00	0.00
43	a	0.03	0.00	....	0.00	0.00	0.01	0.08	0.79	0.08	0.01
	b	0.03	0.00	....	0.00	0.00	0.00	0.13	0.71	0.13	0.00
	c	0.03	0.00	....	0.00	0.00	0.00	0.13	0.71	0.13	0.00
	d	1.00	0.00	....	0.00	0.00	0.00	0.00	0.00	0.00	0.00
44	a	0.03	0.00	....	0.00	0.00	0.00	0.00	0.05	0.87	0.05
	b	0.03	0.00	....	0.00	0.00	0.00	0.00	0.05	0.87	0.05
	c	0.03	0.00	....	0.00	0.00	0.00	0.00	0.05	0.87	0.05
	d	1.00	0.00	....	0.00	0.00	0.00	0.00	0.00	0.00	0.00
45	a	0.03	0.00	....	0.00	0.00	0.00	0.00	0.05	0.10	0.82
	b	0.03	0.00	....	0.00	0.00	0.00	0.00	0.05	0.10	0.82
	c	0.03	0.00	....	0.00	0.00	0.00	0.00	0.05	0.10	0.82
	d	1.00	0.00	....	0.00	0.00	0.00	0.00	0.00	0.00	0.00

a = no management; b = $400/sq km; c = $800/sq km; d = clearcut

Table A.3: Variances Used in Stochastic Transition Matrix

Periods	Strategy	Variance
0-5	1	29
	2	36
	3	42
	4	0
6-15	1	32
	2	39
	3	45
	4	0
16-25	1	36
	2	42
	3	48
	4	0
26-35	1	39
	2	45
	3	51
	4	0
36-45	1	42
	2	48
	3	54
	4	0

where strategy 1 = no management
strategy 2 = $400 / square km / 3 yr period
strategy 3 = $800 / square km / 3 yr period
strategy 4 = clear cut